Math 3

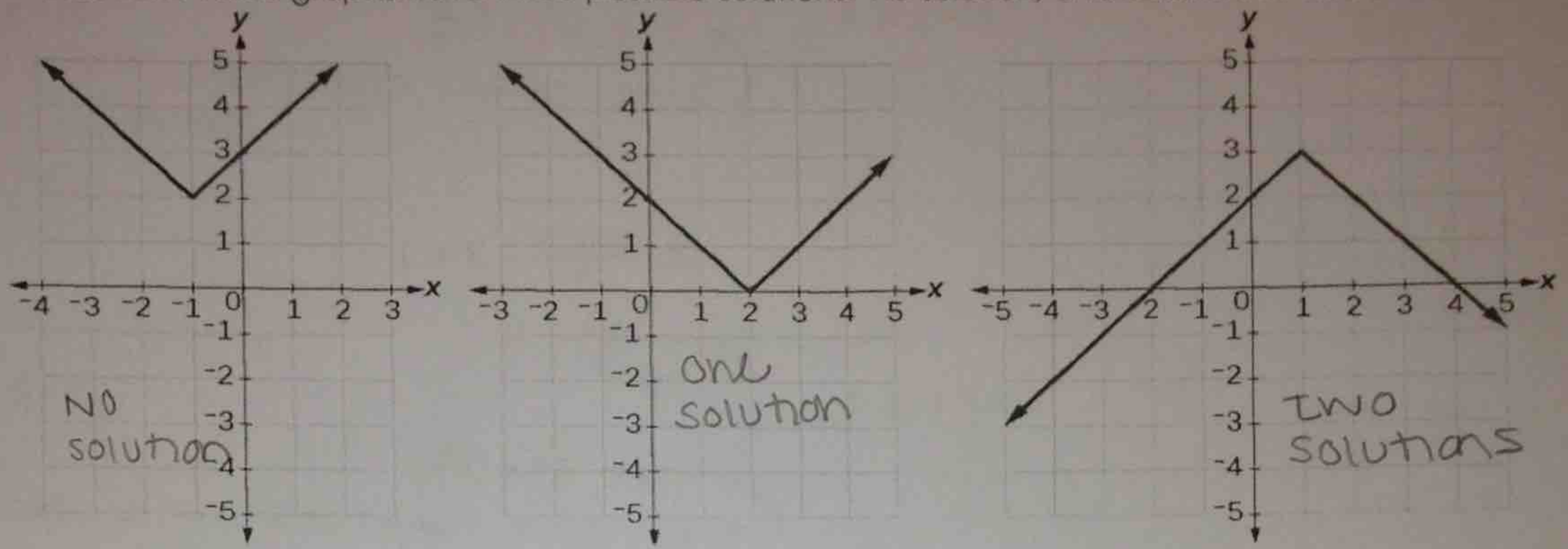
1.6 Absolute Value Equations

Unit 1

SWBAT solve absolute value equations and check solutions using substitution.

Absolute Value: The distance away from something

Absolute value graphs have three possible solutions: no solution, one solution, or two solutions.



(a) (b) (c)

Solving absolute value equations by hand is almost the exact same as solving regular equations with one major difference. In most cases you have *2 solutions.*

Example: $|x| = 5$

We know that when $x = 5$, $|5|$ will also equal 5, but it is also true that $|-5|$ will equal 5. So, for $|x| = 5$, $x = \{-5, 5\}$. They both work.

How to Solve Absolute Value Equations

1. Isolate the absolute value.

NOTE: *Never distribute into the absolute value bars!*

2. Split the equation into two, with one positive and one negative.
3. Check your solution by substituting your answer(s) into the original problem!

Example 1: Solve $|2x+6| - 3 = 13$

$|2x+6| = 16$

$2x+6 = 16$ $\quad$ $2x+6 = -16$

$2x = 10$ $\quad$ $2x = -22$

$x = 5$ $\quad$ $x = -11$

$\{5, -11\}$

Check:

$|2(5)+6| - 3 = 13$

$|16| - 3 = 13$

$16 - 3 = 13$

$13 = 13$ ✓

$|2(-11)+6| - 3 = 13$

$|-16| - 3 = 13$

$16 - 3 = 13$

$13 = 13$ ✓

Example 2: Solve $4|5x-10| + 23 = 3$

$4|5x-10| + 23 = 3$

$4|5x-10| = -20$

$|5x-10| = -5$

$5x-10 = -5$ $\quad$ $5x-10 = 5$

$5x = 5$ $\quad$ $5x = 15$

$x = 1$ $\quad$ $x = 3$

NO solution $\emptyset$

Check:

$4|5(1)-10| + 23 = 3$

$4|-5| + 23 = 3$

$4(5) + 23 = 3$

$20 + 23 = 3$

$43 \neq 3$

$4|5(3)-10| + 23 = 3$

$4|5| + 23 = 3$

$4(5) + 23 = 3$

$20 + 23 = 3$

$43 \neq 3$

Isolating absolute value equations is similar to isolating an equation for x.

Regular Equations

1a) $5x + 9 = 144$

$5x = 135$

$x = 27$

2a) $\frac{x}{7} - 3 = 1$

$\frac{x}{7} = 4$

$x = 28$

3a) $\frac{2}{3}x - 11 = -3$

$\frac{2}{3}x = 8$

$x = 8\left(\frac{3}{2}\right)$

$x = 12$

4a) $\frac{4x-5}{3} = 9$

$4x - 5 = 27$

$4x = 32$

$x = 8$

Absolute Value Equations

1b) $5|3x - 6| + 9 = 144$

$5|3x-6| = 135$

$|3x-6| = 27$

Then split equations and solve!

2b) $\frac{|12x-8|}{7} - 3 = 1$

$\frac{|12x-8|}{7} = 4$

$|12x-8| = 28$

Then split equations and solve!

3b) $\frac{2}{3}|2x - 10| - 11 = -3$

$\frac{2}{3}|2x-10| = 8$

$|2x-10| = 8(3/2)$

$|2x-10| = 12$

Then split equations and solve!

4b) $\frac{4|8x-16|-5}{3} = 9$

$4|8x-16| - 5 = 27$

$4|8x-16| = 32$

$|8x-16| = 8$

Then split equations and solve!

Example 3: $|x+5| = 3x-7$

$x+5 = 3x-7$

$-2x+5 = -7$

$-2x = -12$

$x = 6$

$x+5 = -3x+7$

$4x+5 = 7$

$4x = 2$

$x = 1/2$

$\{6\}$

check: $|6+5| = 3(6)-7$

$11 = 18-7$

$11 = 11$ ✓

check: $|0.5+5| = 3(0.5)-7$

$5.5 = 1.5-7$

$5.5 \neq -5.5$

You Try! $|2t-3| = 3t-2$

$2x-3 = 3x-2$

$-1x-3 = -2$

$-1x = 1$

$x = -1$

check:

$|2(-1)-3| = 3(-1)-2$

$|-2-3| = -3-2$

$5 \neq -5$

$2x-3 = -3x+2$

$5x-3 = 2$

$5x = 5$

$x = 1$

$\{1\}$

$|2(1)-3| = 3(1)-2$

$|-1| = 3-2$

$1 = 1$ ✓